SOME APPROACHES TO THE RÍO DE LA PLATA MODELING

Pablo M. Jacovkis

Departamento de Computación and Instituto de Cálculo
Facultad de Ciencias Exactas y Naturales
Universidad de Buenos Aires
Ciudad Universitaria, 1428 Buenos Aires, Argentina

Abstract

The Río de la Plata can be, has been, and must be, modeled from different viewpoints, with separate or integrated models. On the one hand, it is a typical region for a hydrodynamic two dimensional shallow water modeling; on the other hand, several phenomena related to water quality, dispersion of pollutants, erosion, sedimentation and dredging suggest the formulation of more involved models, including chemical modeling. Besides, some technical problems appear, that have their own interest: as the water level changes, the boundaries change also, and sometimes the changes can not be neglected; and, for a complete model of the Río de la Plata, the outer boundary of the river is an open boundary, and it is not easy to get rid of the phenomenon of numerical waves reflection; for smaller models, with more open boundaries, the problem worsens. In this paper we shall describe some of the different possible approaches to the Río de la Plata modeling, their characteristics, and the feasibility of their integration in one global model.

1. The Río de la Plata: a general (from an Argentinean point of view) approach

It is very difficult to underestimate the social, historic, economic and political importance of the Río de la Plata. It is the entrance to the Plata basin, composed of the Paraná River, the Uruguay River, and their tributaries; its main port, Buenos Aires, was the only outlet of a vast hinterland, and its importance forced the Spanish Kings to create first the city of Colonia, to assure their domain over both bands of it, and then (with its name) a Viceroyalty with enough authority to confront the Portuguese from Brazil. To avoid that the Republic created after the independence from Spain continue to hold both bands, the Portuguese invaded the East Band, and, after the war between Argentina and Brazil (just independent from Portugal) that followed, Britain maneuvered to guarantee the independence of Uruguay. After the consolidation of the organization of Argentina, its harbor, Buenos Aires, was practically the only port in the country, and the control of its tolls almost divided the new country and provoked civil wars (only with the defeat of the Indians around 1880 other ports, in southern Buenos Aires Province and in Patagonia, developed). Practically all the millions of immigrants that arrived in the new country during the second half of the nineteenth century and the first half of the twentieth landed at Buenos Aires, and most Argentineans live in its hinterland. Almost all the naval commerce of Argentina uses the Buenos Aires port, or the ports on the navigable waterway through the Paraná River, mainly Rosario and Santa Fe. The Río de la Plata currently suffers from many problems: on the one hand, suspended material transported from upstream Paraná River originates sediments and solid discharges that force a constant dredging, to allow navigation; on the other hand, the permissiveness and lack of control of industrial pollution during the last fifty years have originated a grave pollution problem: fish are endangered, swimming is dangerous and prohibited, and many tourist activities cannot be performed.

Moreover, the population do not and can not use the Río de la Plata as they should, were it clean. The contrast with Montevideo is striking. And, last but not least, floods during a certain type of storm ("sudestadas", caused by southeastern winds) are responsible for important losses of human lives and property. A future plan for solving the above-mentioned problems of the Río de la Plata may take into account, besides, two important controversial works proposed: a bridge across the Río de la Plata, uniting Argentina and Uruguay (one end should be in or near Buenos Aires, the other in or near Colonia) and the creation of an island to put there the new airport of Buenos Aires, the closest one being dangerous, because is too close to downtown, the farthest one (Ezeiza) being too far away.

Evidently, mathematical models are nowadays a very useful and rather cheap mean to study all these subjects. Several different models can be used, and sometimes integrated in global models. In the following we shall describe some different useful models applicable to the Río de la Plata modeling.

2. Two-dimensional shallow water models

The Río de la Plata (see Figure 1) is clearly two-dimensional and "shallow". So it can be assumed that the equations that govern the unsteady two-dimensional shallow water hydrodynamic flow with free surface are apt to model it. These equations are

$$\partial (hU)/\partial t + \partial (hU^2)/\partial x + \partial (hUV)/\partial y - fhV + gh\partial Z/\partial x + c_f U\sqrt{(U^2 + V^2)} = 0 , \qquad (1)$$

$$\partial (hV)/\partial t + \partial (hUV)/\partial x + \partial (hV^2)/\partial y + fhU + gh\partial Z/\partial y + c_f V\sqrt{(U^2 + V^2)} = 0 , \qquad (2)$$

$$\partial h/\partial t + \partial (hU)/\partial x + \partial (hV)/\partial y = 0 , \qquad (3)$$

where x and y are the horizontal Cartesian axes, t is the time, U and V indicate the velocities in the x- and y-directions, h is the height from the bed level or total water depth, c_f is a friction coefficient, that must be calibrated, and f is the Coriolis parameter. Equations (1) and (2) are the momentum equations, and equation (3) is the conservation-of-mass equation; they may be consulted for instance in Haidvogel and Beckmann [1999]. As in all unsteady models, initial conditions for the three unknown functions $U(x,y,t)$, $V(x,y,t)$, $h(x,y,t)$ are necessary, namely,

$$U(x,y,0) = U_0(x,y) \; ; \; V(x,y,0) = V_0(x,y) \; ; \; h(x,y,0) = h_0(x,y) \; , \qquad (4)$$

where U_0, V_0 and h_0 are known functions. Besides, the Río de la Plata (as well as all estuaries, rivers, lakes, bays, and regions of seas and oceans modeled) is a bounded domain in space, and boundary conditions must be prescribed.

From a mathematical point of view, equations (1-3), with initial conditions (4), and boundary conditions, are a two-dimensional system of quasilinear hyperbolic partial differential equations, written in conservation form; but as no explicit solution of the shallow water equations exist, except in very trivial cases, a numerical solution must be

Figure 1: The Río de la Plata

found. An enormous amount of numerical methods have been studied and implemented in models, since the pioneer work of Leendertse [1967]; many of them are described in Vreugdenhil [1994]. Of course the Río de la Plata has often been modeled; in fact, this author has participated in the preparation and implementation of hydrodynamic models of the Río de la Plata more than twenty years ago (DIGID [1978], Hydrocéano Consultores S.A.-Estudio Gradowcyk y Asociados S.A.T. [1984]); many models have been developed at the Argentinean National Water Agency (INA), formerly National Agency for Water Science and Technology (INCyTH).

It is of course difficult, because many data are necessary, to obtain precise initial conditions for a run of a Río de la Plata model: "warming-up" the model during an initial simulation time usually works. The real problem is calibrating a two-dimensional model, because, if the friction coefficient c_f varies according to the spatial position and to the water level, the number of parameters to calibrate is subject to "the curse of dimensionality"; fortunately, in general we are satisfied with a calibration of fewer values of c_f, because (and this is an advantage over a one-dimensional fluvial model) in this case the geometry of each cross-section is very simple (for instance, for a finite-difference discretization, is rectangular along both axes x, y , see Figure 2) except near a closed boundary, and c_f is a not too complicated function of h; then the number of values of the parameter to be calibrated is linear with respect to the total number of discretization points or model elements; the

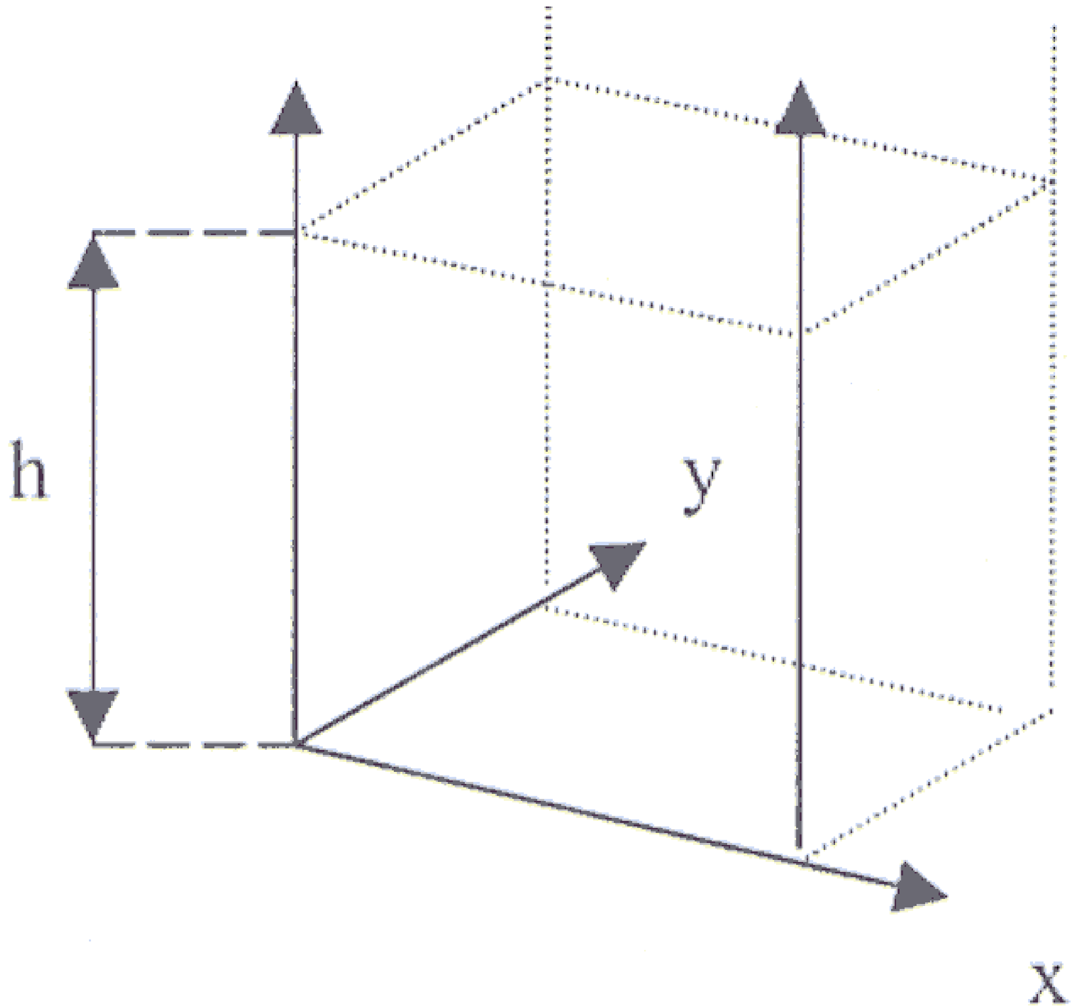

Figure 2: Discretization of a two-dimensional shallow water model

geometry of a one-dimensional model of a natural water course, on the other hand, is more complex, as can be seen in Figure 3, and it is much more difficult to find a suitable algebraic function of *h* that represents it. A possible discretization may be seen in Figure 4.

3. Mobile bed models

A mobile-bed two-dimensional model requires, if the particles only slide and roll, a fourth (conservation of solid mass) equation,

$$(1 - p')\partial e(x,y,t)/\partial t + \partial Q_{bx}(h,U,V)/\partial x + \partial Q_{by}(h,U,V)/\partial y = 0 \, , \qquad (5)$$

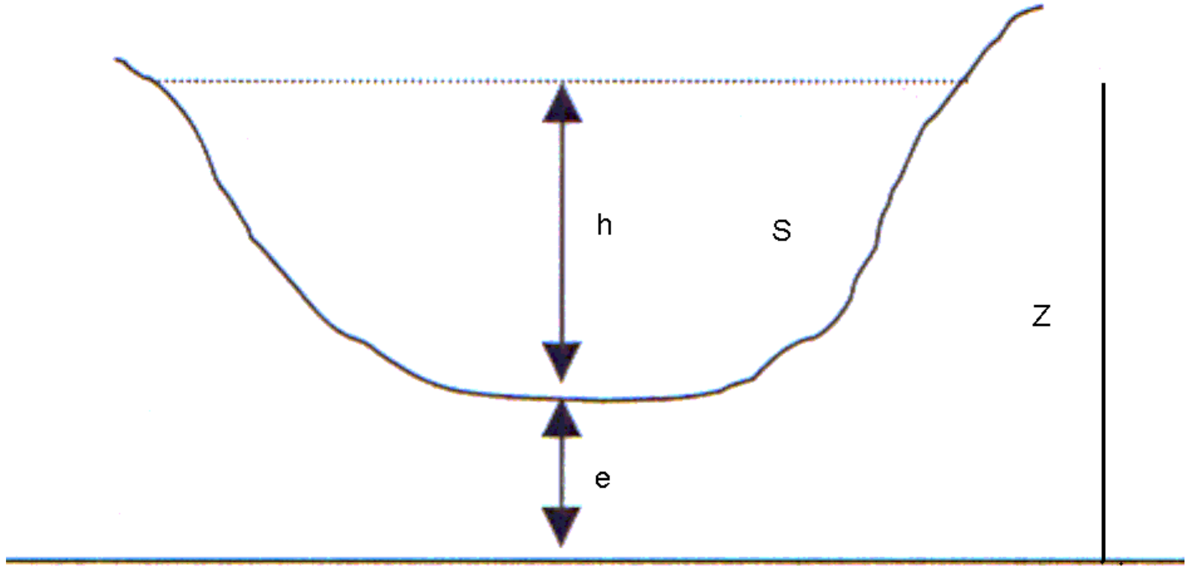

Figure 3: A one-dimensional fluvial cross-section

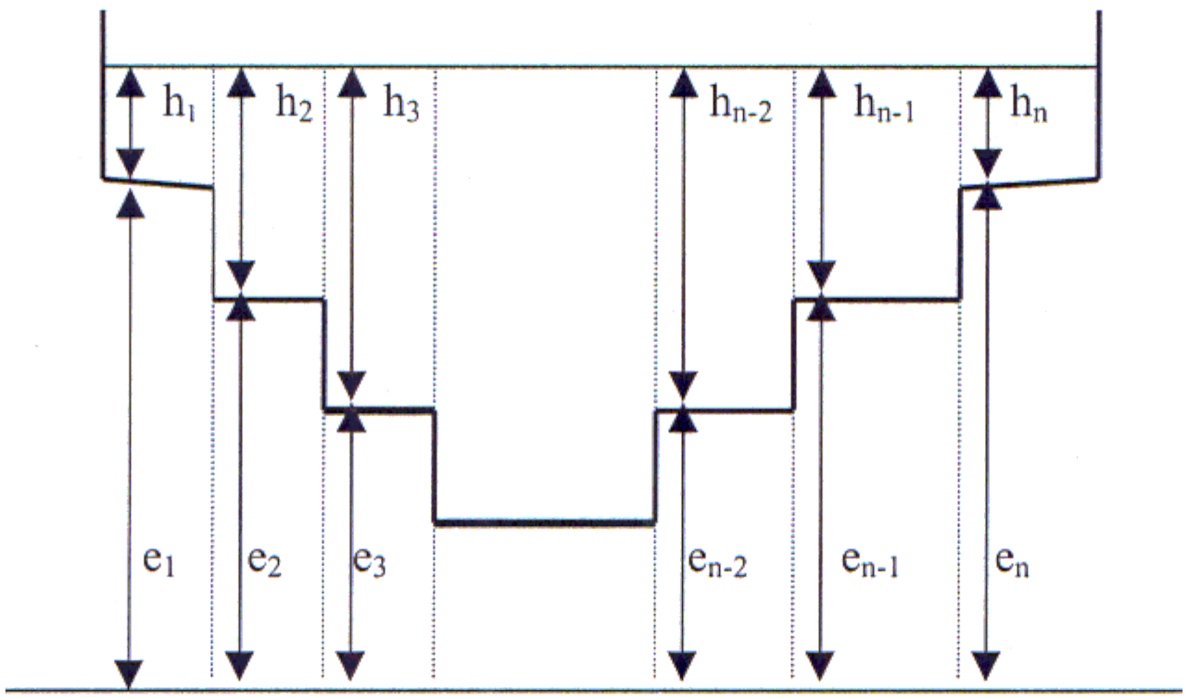

Figure 4: Discretization of a fluvial cross-section

where now Q_{bx} and Q_{by} are the bed-loads, that is, the solid discharges per unit width, in the *x*- and *y*-direction, respectively and p' is the porosity of the material. Formulae for the bed-loads are generally semi-empirical. A careful analysis may be consulted in Wu *et al.* [2000], where $Q_{bx} = \alpha_{bx}Q_b$ and $Q_{by} = \alpha_{by}Q_b$, α_{bx} and α_{by} are the direction cosines, and Q_b is the total bed load. Wu and his coauthors assume that the bed load is in the direction of the bed shear stress so that these cosines are given by the flow calculation. This formula neglects the effect of transverse bed slope, so that it may be used when this slope is small. A more detailed formula may be consulted for instance in van Rijn [1993].

We remark that, although of course a two-dimensional mobile-bed hydrodynamic model is more complex than a one-dimensional one, in a sense the capability of a one-dimensional mobile bed model to really represent the bed behavior is controversial: analyzing Figure 3, for instance, what is really the river bed level? We must generally assume a rectangular cross-section, to be able to define a determined bed level, as in Figure 5. Sometimes this is a reasonable approximation (or we are studying, say, an artificial laboratory channel). But often we must, at least, consider a “quasi-two-dimensional” model, in which we work with parallel streamtubes, each with its own bed level, as may be seen in Gradowczyk and Jacovkis [1984]. To sum up, eventually a two-dimensional model is necessary for an accurate representation of the bed load.

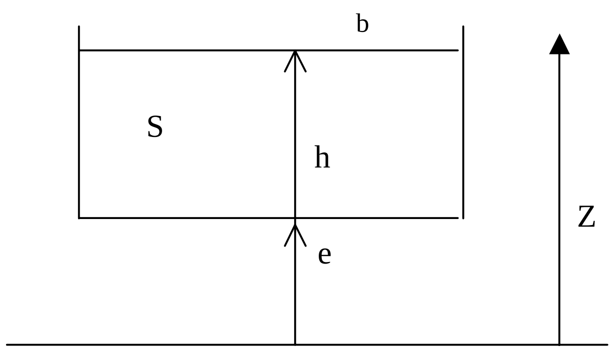

Figure 5: A simple rectangular cross-section

If particles are also transported in suspension, and can be subject to the entrainment (resuspension) or deposition processes, that is, there is exchange of sediment between the bed-load layer and the suspended load region above it, an additional diffusive equation must be included, namely,

$$\partial C/\partial t + \partial (uC)/\partial x + \partial (vC)/\partial y = \partial (\varepsilon_x \, \partial C/\partial x)/\partial x + \partial (\varepsilon_y \, \partial C/\partial y)/\partial y + E_b - D_b \qquad (6)$$

where now $C = C(x,y,t)$ is the local sediment concentration, ε_x and ε_y the sediment mixing coefficients in the *x*- and *y*- directions, respectively, E_s the entrainment (or resuspension) rate from the bed-load layer (upward flux) and D_b the deposition rate (downward sediment flux) of the material. These last two terms form the source or sink, depending on the sign. The deposition rate D_b is proportional to the settling velocity of particles ω_s , $D_b = \omega_s C$, and the entrainment rate E_s has to be introduced with an empirical relation, that can be consulted, for instance, in Celik and Rodi [1988]. In this case the right hand side of equation (5) must also include a source or sink term $D_b - E_s$.

The total load is now the sum of the bed load and the suspended load, namely

$$(1 - p')\partial z_b/\partial t + \partial (hC)/\partial t + \partial Q_{Tx}/\partial x + \partial Q_{Ty}/\partial y = 0 \qquad (7)$$

where Q_{Tx} and Q_{Ty} are the components of the total-load sediment-transport in the *x*- and *y*-directions, respectively.

Evidently, a complete model two-dimensional hydrodynamic model that includes bed load and suspended load is sometimes too involved (currently not necessarily for computational reasons, because two-dimensional models are in general in the range of memory and run-time feasibility for modern computers, but for the amount of data necessary, that sometimes are not easy to obtain); so that some simplifications have been implemented. For instance, a two-dimensional hydrodynamic model with a fixed bed, that does not take into account suspended material, can be used (or perhaps velocity and water depth data already known can be used) and then the results can be used as external values for a sedimentation and mobile bed model, as for example in the report prepared by Estudio Gradowczyk y Asociados S.A.T. and Hydrocéano Consultores S.A. [1982], where this author has participated. Other approach is to consider an integrated model, that includes bed movements, but steady in the liquid velocity and water depth.

4. Water quality models

Equation (6), replacing conveniently the sink and source terms, governs the transport of pollutants. Taking into account that one of the main problems with the Río de la Plata is its pollution, it is no surprise that water quality models for it have been implemented a long time ago. We may mention, for instance, Marshall *et al.* [1978] and Menéndez [1991]. Generalizing to three dimensions, the water quality equation is

$$\partial C/\partial t + u\partial C/\partial x + v\partial C/\partial y + w\partial C/\partial z = \partial (\varepsilon_x \partial C/\partial x)/\partial x + \partial (\varepsilon_y \partial C/\partial y)/\partial y + \partial (\varepsilon_z \partial C/\partial z)/\partial z + r(C,p) \qquad (8)$$

where *w* is now the velocity in the vertical direction *z*, ε_x , ε_y and ε_z are the turbulent or molecular diffusion coefficients, and *r*, function of the concentration and the parameters *p* to be calibrated, represents the conversion rates of state variables due to conversion processes. See for instance Shanehan *et al.* [2000].

5. Analysis of open boundaries

For a shallow water two-dimensional model we have in general three types of boundaries: closed boundary (the boundary condition is normal velocity nil), open boundary, and influx of one-dimensional flows (from one-dimensional fluvial models), represented by a linear combination of Dirac deltas at each time. For the Río de la Plata, these are of course the inflows of the Uruguay River and of the several Paraná delta watercourses, that may be modeled separately. In fact, this has been done for fluvial basins (Jacovkis [1989]), and for fluvial deltas (Jacovkis [1990]). The theoretical analysis of boundary conditions may be seen in Jacovkis [1991]. Other tributaries may be included, for instance the Riachuelo. Boundary conditions are more involved than for one-dimensional problems, because in an open boundary usually there exists the phenomenon of numerical reflection of waves, and it is not easy to get rid of it (see Enquist and Majda [1977]): in one-dimensional models, there are less data (one for open boundary); but in any study of the Río de la Plata it is impossible to avoid the outer open boundary.

Besides, when the boundary of a two-dimensional model is closed, that is, when the boundary is the coast, part of the coast may dry up at low water height, and be flooded at high water height, and this phenomenon means that the boundary is moving. In the Río de la Plata, this situation is regretfully common when sudestadas occur. From a theoretical point of view, some difficulties appear related with the treatment of this situation; from the numerical point of view, several ad-hoc schemes have been applied. We may for instance mention Johns *et al.* [1982] and Gopalakrishnan [1989].

If the model includes mobile bed and/or water quality aspects, boundary conditions can be more involved (for complete integrated models) or simpler (when one restricts the analysis to the mobile bed or water quality equations).

6. Dredging models and technicalities

One of the main problems that affect the Río de la Plata is the continuous sedimentation of suspended material carried by the Paraná River. Taking into account the navigation through the Río de la Plata to Buenos Aires, the main port in Argentina, and upstream through the Paraná River to Rosario and Santa Fe, it is of paramount importance to conveniently dredge the navigation channels in the Río de la Plata, and in the Paraná River. In fact, the freight that ships can store depends on the depth of the dredging, so that the correct dredging of the channels has a strong impact in the cost of goods transported. A general dredging model has to assess the balance between the dredged and settled material: usually, the volume of material to be dredged is a boundary condition, that is, the dredging policy is known for each simulation, and after dredging is interrupted or finished the model must compute the time necessary for the sedimentation to nullify its effects. A general model may be used, taking into account that dredging is applied not to the complete two-dimensional region modeled, but only to specially chosen channels (usually for navigation purposes), so that two possible simplified approaches are the following: a general two-dimensional model is run, without dredging, that computes sedimentation and erosion, and then, only over the channels to be dredged, dredging volume is imposed to the results, or a one-dimensional model including only a navigation channel is prepared, that considers only velocities, water depths and bed movements along the navigation channel, modifying the bed movements according to dredging. Usually one tries to experimentally find the best dredging policy, that is, the policy that maintain the navigation channel operative with the least dredging activity. In the past decade, this author has participated in dredging studies of the Río de la Plata and the Paraná River using the one-dimensional approach, plus a special model that locally computes sedimentation and erosion for several types of channels (see Deloitte & Co. *et al.* [1999]).

A reasonable policy of maintenance dredging establishes two bed level values, e_{max} and e_{min}. When the bed level reaches e_{max}, dredging begins until the value e_{min}. This approach allows us to simulate, for extensive periods, the embankments that would be produced in a navigable channel without dredging. So the evolution of volumes to sediment can be forecast when solid discharged transported in suspension and the hydraulic and sedimentological data are known.

7. Conclusions

Although often a simplified solution is enough for many purposes, as has been described above, and this simplified solution obviously requires cheaper computational equipment (sometimes several order of magnitude cheaper), one does not has to think that if a complete two-dimensional (or perhaps three-dimensional) model is necessary the computational expenses must compulsorily be very expensive. One thinks in a supercomputer, maybe, but often it is not necessary to buy – or rent – a supercomputer. More and more clusters of personal computers may (although not in all cases) replace supercomputers because, on the one hand, they have a better performance, and, on the other hand, they cost much less and have less maintenance costs. Sometimes a very complete two-dimensional model can be efficiently run in a not too complex cluster, using much less money than was originally thought. Sometimes the real cost in not in the computational equipment, but in the feasibility of obtaining complete and correct data, especially in underdeveloped countries. One can distinguish current trends visiting the web site www.top500.org where the 500 faster computers are shown.

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